Homework

In a city of 1 million inhabitants there are 100 known terrorists and 999.900 non-terrorists.

The prior probability of one random inhabitant of the city being a terrorist is thus 0.0001 and the prior probability of a random inhabitant being a non-terrorist is 0.9999. In an attempt to catch the terrorists, the city installs a surveillance camera with automatic facial recognition software. The software has two failure rates of 1%:

- if the camera sees a terrorist, it will ring a bell 99% of the time, and mistakenly fail to ring it 1% of the time (in other words, the false-negative rate is 1%).
- if the camera sees a non-terrorist, it will not ring the bell 99% of the time, but it will mistakenly ring it 1% of the time (the false-positive rate is 1%).

So, the failure rate of the camera is always 1%.

Suppose somebody triggers the alarm. What is the chance that is a terrorist?

The base-rate fallacy

Imagine that the city's entire population of one million people pass in front of the camera. About 99 of the 100 terrorists will trigger the alarm— and so will about 9.999 of the 999.900 non-terrorists. Therefore, about 10,098 people will trigger the alarm, among which about 99 will be terrorists. So the probability that a person triggering the alarm is actually a terrorist is only about 99 in 10.098, which is less than 1%, and very very far below the initial guess 99% that many people does.

Why this error?
The "**base rate fallacy**" is the incorrect assumption that:

$$P(terrorist / bell) \stackrel{?}{=} P(bell / terrorist) = 99\%$$

The base-rate fallacy

To estimate correctly the probability you must use the Bayes theorem

$$P(\text{terrorist}|\text{bell}) = \frac{P(\text{bell}|\text{terrorist})P(\text{terrorist})}{P(\text{bell})} = \frac{P(\text{bell}|\text{terrorist}) \times P(\text{terrorist})}{P(\text{bell}|\text{terrorist}) \times P(\text{terrorist}) + P(\text{bell}|\text{nonterrorist}) \times P(\text{nonterrorist})}$$

$$= \frac{0.99 \cdot (100/1,000,000)}{\frac{0.99 \cdot 100}{1,000,000} + \frac{0.01 \cdot 999,900}{1,000,000}}$$

$$= 1/102 \approx 1\%$$

The base-rate fallacy

Base rate fallacy, also called **base rate neglect** or **base rate bias**, is a common error in human thinking (Maya Bar-Hillel, Acta Psychologica, 1980).

The base-rate fallacy is people's tendency to ignore base rates in favor of, e.g., individuating information (when such is available), rather than integrate the two (using Bayes theorem).

Information is deemed more relevant when it relates more specifically to a judged target case.

The base-rate fallacy: another example

Two cab companies operate in a given city, the Blue and the Green (according to the colour of cab they run).
Eighty-five percent of the cabs in the city are Blue, and the remaining 15% are Green.

A cab was involved in a hit-and-run accident at night.

A witness later identified the cab as a Green cab. The court tested the witness' ability to distinguish between Blue and Green cabs under night time visibility conditions. It found that the witness was able to identify each colour correctly about 80% of the time, but confused it with the other colour about 20% of the time.

What do you think are the chances that the errant cab was indeed Green, as the witness claimed?

The base-rate fallacy: another example

This is a paradigmatic Bayesian inference problem. It contains two kinds of information. One is in the form of **background data** on the colour distribution of cabs in the city, called base-rate information (prior probabilities). The second, rendered by the witness, relates specifically to the cab in question, and is here called indicant or diagnostic information.

If your initial inclination is to be 80% sure that the witness' testimony of Green is in fact reliable, then you are exhibiting the base-rate fallacy, namely, the fallacy of allowing indicators to dominate base rates in your probability assessments.

The right answer comes from the Bayes theorem:

$$P(G/g) = P(g/G)\, P(G) \,/\, P(g) = P(g/G)\, P(G) \,/\, P(g/G)\, P(G) + P(g/B)\, P(B) =$$

$$= 0.8 \times 0.15 \,/\, 0.8 \times 0.15 + 0.2 \times 0.85 = 0.12 \,/\, 0.29 = 41\%$$

Where g denotes the testimony that the cab was green.