Addendum to: Diversity in classifier ensembles: fertile concept or dead end?

Proc. 11th Int. Workshop on Multiple Classifier Systems,

Springer LNCS Vol. 7872, pp. 37-48, 2013

Luca Didaci, Giorgio Fumera, and Fabio Roli

Dept. of Electrical and Electronic Engineering, University of Cagliari
Piazza d'Armi, 09123 Cagliari, Italy
email: {luca.didaci, fumera, roli}@diee.unica.it
WWW home page: http://prag.diee.unica.it/en

In this online addendum we report two derivations for which there was no room in the paper. To avoid ambiguities, references to equations of the original paper will be written as (A.n). To obtain these derivations we follow a simpler procedure than the one outlined in the paper.

A Derivation of the ambiguity-like decomposition (A.12)

In this section we derive the ambiguity-like decomposition (A.12) of the 0/1-loss of a classifier ensemble. We want to write the expected error of an ensemble of N classifiers, with respect to the random variables D and $Y|\mathbf{x}$, as the sum of the average expected error of the individual classifiers and a term that plays the role of the ambiguity term in the ambiguity decomposition (A.6) for regression problems. Denoting such a term by A:

$$E_{D,Y|\mathbf{x}}[e_{\text{ens}}(\mathbf{x}, Y; D)] = E_{D,Y|\mathbf{x}}[\overline{e}(\mathbf{x}, Y; D)] + A \ . \tag{1}$$

We thus compute A as:

$$A = E_{D,Y|\mathbf{x}}[e_{\text{ens}}(\mathbf{x}, Y; D) - \overline{e}(\mathbf{x}, Y; D)] \ . \tag{2}$$

The above expectation can be immediately rewritten in terms of probabilities:

$$\begin{aligned} A &= E_{D,Y|\mathbf{x}}\left[e_{\text{ens}}(\mathbf{x}, Y; D) - \frac{1}{N}\sum_j e_{f_j}(\mathbf{x}, Y; D)\right] = \\ &= \left(1 - P_{D,Y|\mathbf{x}}[f_{\text{ens}} = Y|\mathbf{x}]\right) - \frac{1}{N}\sum_j \left(1 - P_{D,Y|\mathbf{x}}[f_j = Y|\mathbf{x}]\right) = \\ &= -\frac{1}{N}\sum_j \left(P_{D,Y|\mathbf{x}}[f_{\text{ens}} = Y|\mathbf{x}] - P_{D,Y|\mathbf{x}}[f_j = Y|\mathbf{x}]\right) \ . \end{aligned} \tag{3}$$

The value of A can also be rewritten by separately computing the expectations with respect to D and $Y|\mathbf{x}$, since they are independent random variables:

$$\begin{aligned}
A &= E_{D,Y|\mathbf{x}}[e_{\text{ens}}(\mathbf{x},Y;D) - \overline{e}(\mathbf{x},Y;D)] = \\
&= E_{D,Y|\mathbf{x}}\left[e_{\text{ens}}(\mathbf{x},Y;D) - \frac{1}{N}\sum_j e_{f_j}(\mathbf{x},Y;D)\right] = \\
&= E_{Y|\mathbf{x}}E_D\left[e_{\text{ens}}(\mathbf{x},Y;D) - \frac{1}{N}\sum_j e_{f_j}(\mathbf{x},Y;D)\right] = \\
&= \sum_{y_i} P[y_i]E_D\left[e_{\text{ens}}(\mathbf{x},y_i;D) - \frac{1}{N}\sum_j e_{f_j}(\mathbf{x},y_i;D)\right] = \\
&= \sum_{y_i} P[y_i]\left[1 - \hat{P}_{\text{ens}}[y_i] - \frac{1}{N}\sum_j \left(1 - \hat{P}_j[y_i]\right)\right] = \\
&= -\sum_{y_i} P[y_i]\frac{1}{N}\sum_j \left(\hat{P}_{\text{ens}}[y_i] - \hat{P}_j[y_i]\right)
\end{aligned} \tag{4}$$

Derivation of the "covariance" term in (A.10)

We denote the term $b+v$ of (A.10) by C. It plays a role analogous to the covariance term in the bias-variance-covariance decomposition of regression error (A.5). To compute C we exploit the ambiguity-like decomposition (1), and rewrite it by replacing the expected errors with the corresponding Kohavi-Wolpert bias-variance decomposition (A.8):

$$\begin{aligned}
bias_{f_{\text{ens}}} + var_{f_{\text{ens}}} + noise &= \overline{bias} + \overline{var} + A + noise = \\
&= \overline{bias} + \frac{1}{N}\overline{var} + \frac{N-1}{N}\overline{var} + A + noise\ .
\end{aligned} \tag{5}$$

From (5) and (A.10) we obtain:

$$C = \frac{N-1}{N}\overline{var} + A\ . \tag{6}$$

Replacing $\overline{var}$ with $\frac{1}{N}\sum_j var_{f_j}$, var_{f_j} with its definition (A.7), and A with (4), we finally obtain:

$$C = \frac{N-1}{N^2}\sum_j \frac{1}{2}\left(1 - \sum_{y_i}\hat{P}[y_i]^2\right) - \sum_{y_i} P[y_i]\frac{1}{N}\sum_j \left(\hat{P}_{\text{ens}}[y_i] - \hat{P}_j[y_i]\right)\ . \tag{7}$$